Restructuring Mathematics Teacher Education:
The Evolution of an Innovative Preservice Program in Russia

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Political and social changes in Russia have had an impact on all institutions of Russian society, including the educational system (Kerr, 1990, 1991). The advent of *glasnost* has promoted free inquiry and independent and critical thinking in government, business, industry, and across the curriculum. These features of democracy have been traditionally emphasized by mathematics teachers of gifted and talented students in the former Soviet Union (Toom, 1993)—individuals studying mathematics, an apolitical subject devoid of Marxist-Lenist ideology, were not censored or monitored by the Communists (Curcio, 1986; Questionnaire, 1993).

As standards in mathematics education are developing in Russia, a current concern is to provide an appropriate mathematics curriculum containing these features for **all** students (Dorofeyev *et al.*, 1993). A way to begin to accomplish this is to improve the

preparation of preservice teachers (Kerr, 1989).

In Russia, preservice teacher preparation for the primary grades (*i.e.*, kindergarten through grade 3) is conducted by educators in pedagogical academies. These academies offer courses in Russian language, literature, mathematics, science, history, and teaching methods. They are analogous to normal schools and are viewed "as an advanced extension of secondary education" (Kerr, 1991, p. 333). Preservice teacher education for the secondary grades (*i.e.*, grades 4 through 11) is conducted by educators in pedagogical universities, which until recently were called "institutes." The teacher preparation program in the pedagogical universities contains three components: discipline specialty, psychology and sociology, and pedagogy (Kerr, 1991, p. 334). Pedagogical universities offer discipline specialty programs in mathematics education, foreign languages, art education, physical education, and so on. Traditionally, these programs have been very "rigorous and comprehensive"—in particular, preservice mathematics students are "exposed to more advanced topics in mathematics than their American counterparts (Ferrucci & Evans, 1993, p. 89).

Recognizing that reaching **all** students in secondary mathematics classes will require alternative instructional strategies beyond merely lecturing, one of the pedagogical universities in St. Petersburg has been examining ways to restructure and strengthen the teacher preparation program.[1] Similar to preparing teachers in the United States, in addition to offering preservice teachers alternative instructional strategies teacher educators at Russia State Pedagogical University (RSPU) believe that teachers of mathematics need "a deep understanding of the mathematics of the school curriculum and how it fits within the discipline of mathematics" (NCTM, 1991, p. 134).

Several limitations of the traditional centralized system of teacher preparation in the former Soviet Union have provided the impetus for restructuring the teacher preparation program at RSPU. First, the traditional method of teaching, that is lecturing, is inadequate in reaching all students. Traditionally, only mathematically gifted students were encouraged to pursue the study of mathematics. Currently, to build a competitive economy and workforce, more citizens must become fluent in mathematics. Second, freshmen entering RSPU, who have been exposed to lecturing exclusively in secondary schools, arrive with a very limited view of mathematics and are not adequately prepared to become secondary mathematics teachers capable of reaching **all** students. Thus, the program was restructured.

The program at RSPU is a five-year program which prepares teachers of mathematics for grades five through eleven.[2] One of the main problems the faculty at RSPU needed to address was the relationship between the mathematics needed for future mathematicians and the mathematics needed for prospective teachers. Teachers must be able to explain the underlying meaning of concepts and skills and not just know and utilize the concepts for future reference. The faculty members believe that the students do not understand the mathematics they come with, and

they lack "mathematical culture"—that is, they lack a mathematical way of thinking, unable to reason and analyze problems. The students' accurate use of the language of mathematics is deficient. They do not understand definitions, proofs, and other logical arguments. The students are very algorithmically oriented. (See Appendix One for sample items from a high school final exam. The nature of the problems exemplify the emphasis placed on skills.) At a time when the educational system is becoming more decentralized, RSPU faculty took the initiative to revise the program, addressing this issue dealing with the improvement of teacher preparation. The changes introduced are not typical of preservice mathematics teacher preparation in Russia.

The program that was created is divided into three stages. Stage One consists of two courses taken in the first semester to help the students bridge the gap between the reasoning and mathematics received in high school and the reasoning needed for university level mathematics. One course is an introductory course on algebra and analysis taught ten hours per week. The second course is on geometry and is taught five hours per week. Stage Two lasts the next four semesters and includes the "basic" mathematical content, covering topics in algebra, analysis, and geometry (see Appendix Two for a list of topics and time spent). Stage Three, covering the last two and one-half years, concentrates on methodology and pedagogy, including practice teaching. At this stage, students have the opportunity to select a mentor from the mathematics department to study a particular area of mathematics more intensely. The stages have some very interesting features which will be described in this article.

Stage One

The introductory course on algebra and analysis was created to address the problem relating to a lack of reasoning abilities and to address the differences between the mathematics needed by teachers versus the mathematics needed by mathematicians. Although the mathematics students learn in high school includes analysis, it is usually quite algorithmic in nature. Students are usually asked to do something and are not asked to explain why they do it, and as teachers, they need to know the "why." Also, the courses they take in Stage Two during the next four semesters are taken with mathematics majors, and the courses usually have a different emphasis as compared with how prospective teachers are prepared.

The primary focus of the introductory course is to help students to "think mathematically." The content of the course, which has been evolving since 1988, is secondary to the development of mathematical thinking. The content is the vehicle for the development of the mathematical culture needed for success. Currently, it consists of: (1) the beginnings of logic; (2) the language of sets, which is not in the secondary curriculum now; (3) solving and graphing equations and inequalities, including proofs and counter examples; (4) mappings and functions

and invariance under certain mappings; (5) the beginnings of combinatorics; and (6) mathematical induction.

The course, characterized by recitation rather than lecture, is taught in a variety of formats, but usually involves a lot of discussion by students in large and small groups. No textbook is used for the course, but the faculty who teach it have prepared a manuscript entitled, *Introduction to Algebra and Mathematical Analysis, Problem Book 1: Statements, Predicates, Sets*. This book is the main source of problems presented throughout the course as students work in teams. Upon completion of the course, a student takes an exam with about five or six lengthy and complex questions, which require a thorough explanation of his or her thinking. The passing score usually accepted is about 35 percent correct. Only about 70-to-75 percent of the students pass the exam, in spite of the fact that the student has three attempts (with two-week intervals between exams) to pass it. (See Appendix Three for a copy of one of these exams.)

During this stage, students also take a geometry course for five hours a week covering ideas from coordinate and vector geometry. At the end of the semester, students must take an oral exam. Only those students passing both the oral exam and the exam for the introductory course are allowed to move to Stage Two of the program. It should be noted that students are taking 15 hours of mathematics their first semester as well as a computer science course which is taught six hours a week.

Stage Two

Stage Two consists of the basic mathematics courses taken by all mathematics majors. Analysis, consisting of differential and integral calculus, is taught eight hours a week; algebra, consisting of linear and modern algebra, is taught four hours a week; and, geometry, consisting of Euclidean and non-Euclidean geometries, is taught four hours a week for two semesters. A student takes between 14 to 18 hours of mathematics each of the four semesters comprising this stage (see Appendix Two). An additional six hours per week of computer science is taught each semester. Students must also take two basic pedagogical and two basic psychology courses during these four semesters.

The pedagogy courses include an analysis of student-teacher relationships, alternative approaches to the lecture method, ideas for assigning projects and tasks, an examination of Olympiad problems, and ideas for involving students in independent research on historical topics in mathematics. One of the basic pedagogy courses is entitled, "Cooperation of Teachers and Students in Special Mathematics Schools." The psychology courses build on the general work of Lev Vygotsky and Boris Teplov. The psychology of learning and teaching mathematics is based on the work of V.A. Krutetskii. One of the basic psychology courses is entitled, "Psychological Prognosticating in Professional Activity of Mathematics Teachers."

Stage Three

The last two and one-half years constitutes the third and final stage of the program. At this stage students elect to work with a mentor. The mentor working closely with students in this final stage, teaches about 50 percent of the additional mathematics courses for his or her "tutees." Of the courses taught by the mentor, one-fourth of them are devoted to advanced mathematics (*e.g.*, measure theory, Hilbert spaces), and three-fourths are devoted to the mentor's choice (*e.g.*, school mathematics, geometry of the triangle, recursion, Fibonacci numbers). Courses as well as problem-solving seminars (*e.g.*, mathematics Olympiad problems) are conducted. The mentor chooses the style of the presentations—lectures or seminars.

A student's first clinical experience is observation which occurs in the second semester of his or her third year—for a period of three weeks. Practice teaching is done at the fifth and sixth grades, with the first one-and-a-half weeks being a period of observation and the student teacher serving as a teacher's aid. During the last week-and-a-half, the student teachers are engaged in teaching some lessons. In the fourth year, students teach in grades seven through nine for five weeks. And in the fifth year, students teach for eight weeks at the tenth and eleventh grades.

The methods of teaching mathematics present the theoretical basis of teaching, including the analysis of textbooks they use while teaching. This analysis consists of examining the logical structure of texts, how to teach the content, how to organize oral and written work with the class, how to plan lessons, and how to do this in different schools (*i.e.*, mathematics specialty schools and "regular" schools). The mentor works with his or her students on these questions and observes them as they student teach in their assigned classroom.

Closing Comments

One of the most fascinating aspects of studying mathematics education in countries where culture, heritage, and customs are quite different from our own, is that many of the problems we face are universal—trying to reach all students, strengthening the mathematical preparation of future teachers, and building preservice teachers' belief systems that support communicating the structure of the discipline accurately. It is only recently that the Russians are attempting to reach all students in mathematics. Developing alternative instructional strategies that go beyond the lecture method will provide preservice teachers with tools for meeting individual needs. Working in small groups and engaging in mathematical discussions model instructional approaches that preservice teachers may use with their students.

Teachers cannot teach what they themselves do not know. The "why" of mathematics forms the basis for developing meaning and understanding. The

RSPU preservice teachers are immersed in a deeper level of understanding mathematics that goes beyond the mechanical execution of algorithms. This will help them teach for mathematical understanding.

Teachers communicate their beliefs about mathehmatics to their students. If teachers have a myopic view of mathematics they will transmit their limited view to their students. The RSPU preservice teachers have the opportunity to explore mathematics more deeply, experiencing a mathematical culture that will expand and broaden their perspective, appreciation. and understanding of the subject.

The RSPU preservice program has several desirable elements that will improve the preparation of secondary mathematics teachers in St. Petersburg. Although we do not advocate adopting programs that are successful in different countries without considering the context and setting in which they exist, we can gain insight into the nature of our common problems and examine possible solutions by sharing ideas and engaging in dialogue, exchanging reactions. We hope that providing information about the RSPU preservice program will encourage mathematics educators to share other innovative approaches to teacher preparation.

Notes

1. The program described in this article was designed by Dean Natalya Stefanova, Professor V. Kobielsky, Professor Alexander Plotkin, and Professor A. Verner at Russia State Pedagogical University, St. Petersburg, Russia.
2. Unlike the 12-year educational system in the United States, the Russian education system spans 11 years.

References

Curcio, Frances R. "The Development of Current Trends in Mathematics Education in Soviet Schools." In *Soviet Politics and Education* . Edited by Frank M. Sorrentino & Frances R. Curcio, pp. 361-372. Lanham, MD: University Press of America.

Dorofeyev, G.V., K.A. Krasnyamskaya, L.V. Kuznetsov, I.A. Luryo, S.S. Minayeva, S.B. Suvorova, A.V. Shevkin, L.O. Denishcheva, M.B. Mindyuk, & L.O. Roslova. *The Provisional State Education Standard: General Secondary Education—Mathematics* (trans. by Bradley Stewart). Moscow, Russia: Ministry of Education of the Russian Federation, 1993.

Ferrucci, Beverly J. & Richard C. Evans. "Mathematics Teacher Education: A Model from Crimea." *Teacher Education Quarterly* 20 (Spring 1993): 85-90.

Kerr, Stephen T. "Reform in Soviet and American Education: Parallels and Contrasts. *Phi Delta Kappan*, 71 (September 1989): 19-28.

Kerr, Stephen T. "Will Glasnost Lead to Perestroika? Directions of Educational Reform in the USSR." *Educational Researcher*, 19 (October 1990): 26-31.

Kerr, Stephen T. "Beyond Dogma: Teacher Education in the USSR." *Journal of Teacher Education*. 42 (November-December 1991): 332-349.

National Council of Teachers of Mathematics. *Professional Standards for Teaching Mathematics*. Reston, VA: The Council, 1991.

"Questionnaire on Mathematics in the Former Soviet Union." *Notices of the American*

Mathematical Society, 40 (February 1993): 132-137.
Toom, Andrei. "A Russian Teacher in America." *Journal of Mathematical Behavior*, 12 (June 1993): 117-139.

Appendix One

Sample Items from a Final Exam on High School Algebra and Analysis

1. Solve the equation $\frac{2x^2 - 5x - 3}{3^x - 3} = 0$.

2. Solve the inequality $\frac{\log_{0.5}(x^2 - 3)}{\log_{0.5} 3} \leq 0$.

3. Write an equation of the tangent line to the graph of the function $y = x^2 - 3x + 4$ at the point of this graph with abscissa $x_0 = 1$.

4. Find all values of x for which the expression $(\sqrt{6 - x^2 - x}) \tan \frac{\pi x^2}{4}$ exists and is not equal to 0.

5. Find all values of b for which the equation $(x + 4)/x = b$ has no roots.

Appendix Two

Secondary Mathematics Teacher Preparation Program at Russia State Pedagogical University

Stage One

Semester One (16 weeks)

The aim is not to introduce new material but rather to change the students' limited school "point of view" and to bridge between school mathematics and mathematical reasoning. Students learn how to construct mathematical arguments.

A. Introductory Algebra and Analysis (10 hours per week)—Students work in groups, engage in discussions, no lectures, they study together. Content is school mathematics with correct mathematical language. Sets, logic, combinatorics, inequalities, elementary functions
B. Elementary Geometry (5 hours per week)—2 hours lecture, 3 hours seminars
C. Exams in Introductory Algebra and Analysis, and Geometry

Stage Two

	Lecture	Seminar/Recitation
Semester Two (17 weeks)		
Mathematical Analysis	4 hrs	4 hrs
Algebra	2 hrs	2 hrs
Geometry	2 hrs	2 hrs

Stage Two (continued)

	Lecture	Seminar/Recitation	
Informatics	1 hr	1 hr	
Semester Three (15 weeks)			
Analysis	3 hrs	4 hrs	Students also enroll in psychology and pedagogy. These are obligatory.
Algebra	2 hrs	2 hrs	
Geometry	2 hrs	2 hrs	
Informatics	1 hr	1 hr	
Semester Four (17 weeks)			
Analysis	3 hrs	2 hrs	
Algebra	2 hrs	2 hrs	
Geometry	2 hrs	1 hr	
Informatics	1 hr	1 hr	
Probability Theory	4 hrs		
Semester Five (15 weeks)			
Analysis	3 hrs	2 hrs	
Algebra	2 hrs	2 hrs	
Geometry	2 hrs	1 hr	
Methods of Teaching	2 hrs	2 hrs	
Probability Theory	4 hrs		

Stage Three

After the fifth semester, when the basic courses are completed, there is additional training. Students select a faculty mentor to study a particular mathematical content area in more depth. Mentors teach at least one half of the additional work. Other professors are invited to present similar as well as contrasting views.

Appendix Three

Exam Taken by Prospective Teachers Prior to Entering Stage Two of the Program for Teachers of Mathematics, Grades 5 through 11, at Russia State Pedagogical University, St. Petersburg, Russia

Introduction to Algebra and Mathematical Analysis
Exam Two—31 January 1992

1. For the three sets A, B, and X consider the equality statement:
   $A \setminus X = B \cap X$ (*)
   a) Prove that statement (*) is valid if and only if both parts of this equality are empty.
   b) Is it correct that (*) $\Rightarrow A \cap B = \varnothing$?
   c) Let A = (0,1) and B = (2,3).
   Find the interval X such that $X \subset (0,3)$ and (*) is valid.

2. Let f be a number function and $g=f^2$.
   a) Prove that for f to be injective it is sufficient that g is injective.
   b) Is it correct that for f to be injective g necessarily should be injective?
   c) Prove that if f is injective and $f \geq 0$ then g is injective.

3. Let ρ be a binary relation in $\mathbb{R}$: $(x,y) \in \rho \Leftrightarrow (|x-2y| > 2) \vee (|x-y| < 1)$.
   a) Graph $M = \{(x,y): x \geq y \text{ and } (x,y) \notin \rho\}$.
   b) Is ρ reflexive?
   c) Is ρ anti-symmetric?

4. Let $u(x) = x^2 + 2x \quad v(x) = |x| \quad w(x) = x/(x-3)$.
   a) Graph $f = w \blacklozenge u \blacklozenge v$.
   b) Graph $g = v \blacklozenge w \blacklozenge u$.
   c) Let $h = w \blacklozenge v \blacklozenge u$. Find E_h.
   d) For which a does the equation $h(x) = a$ have no solution?
   e) Which of the functions u, v, w, f, g, h have inverses?
   f) Prove that the straight line $y = x$ is not an axis of symmetry of the graph of the function w.

5. Doe s there exist a function g such that:
   a) $\forall x \in \mathbb{R} \quad x^3 = g(1-x^4)$?
   b) $\forall x \in \mathbb{R} \cos x = g(1-x^4)$?

6. Let $a, b, x \in \mathbb{R}$
   a) Disprove the statement: there exists a number a that for any b an x can be found such that $(x-a)(x-b) < 0$.
   b) Is it correct that for any a there exists a certain b, such that for any x, $x>a$ or $x<b$?

7. Let $f(x) = 4\sin^2(4x\frac{\pi}{3}) + 1$.
   a) Find the least positive period for f.
   b) Is it correct that $f([\frac{\pi}{12}, \frac{\pi}{3}]) = f(R)$?
   c) Find any center of symmetry for the graph of function f.

8. Let us consider two inequalities
   (1) $x > x^2$ and $x + h(x) > x^2 + h(x)$
   a) Find a function h such that (1) $\nLeftrightarrow$ (2).
   b) Is it correct that: if (1) $\Leftrightarrow$ (2), then $D_h \supset (0, +\infty)$?
   c) Are the inequalities $\log_2 |3x - 14| \leq 1$ and $-2 \leq 3x - 14 \leq 2$ equivalent?

Note: Ms. Larissa Mezentseff is gratefully acknowledged for translating this examination from Russian into English.

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